Section 9

1. Customers arrive at the infamous doctor’s office according to a Poisson process with rate \( \alpha = 1/10 \) minutes. Careful observation reveals that the doctor does not bother admitting patients until at least three patients are in the waiting room.
   (a) Find the expected waiting time until the first patient is admitted to see the doctor.
   (b) What is the probability that nobody is admitted in the first hour to see the doctor?

2. There is a restaurant for serving great food, but it is filthy and hence not for those with weak stomachs. During rush hour, customers arrive at the restaurant according to a Poisson process of rate \( a = 1/10 \) minutes. Customers peek in the door and with probability \( p \) they enter and eat; with probability \( 1 - p \) they decide the filth is not for them and depart.
   (a) What is the mean and variance of the waiting time between entrances of customers into the restaurant?
   (b) What is the probability that at least one customer eats in the restaurant in the first hour?

3. Green men arrive at the launching pad according to a Poisson process with rate \( \mu = 2 \) while red men arrive according to a Poisson process with rate \( \lambda = 3 \). The space ship leaves when there are 3 men (of either color) at the pad.
   (a) What is the expected time required to fill the space ship?
   (b) What is the probability that the crew are all green?

4. Motor vehicles arrive at a bridge toll gate according to a Poisson process with rate \( \alpha = 2 \) vehicles per minute. The drivers pay tolls of $1, $2 and $5 depending which of the three weight classes their vehicles belong. Assuming the vehicles arriving at the gate belong to class 1, 2 and 3 with probabilities 1/2, 1/3 and 1/6 respectively. Find
   (a) The mean and variance of the amount in dollars collected in any given hour.
   (b) The probability that exactly $3 is collected in each of three consecutive 1-minute intervals (which are specified).
   (c) The probability that the waiting time between the first two $5 vehicles is more than 10 minutes.

5. Incidents of drug usage occur in the bathroom of Harry’s restaurant. The times of these incidents constitute a Poisson process of rate two per hour. Harry is concerned about this because of its possible impact to business. Consequently, Harry visits the bathrooms at times that constitutes a Poisson process of rate one per hour. Assume that this Poisson process is independent of the Poisson process of drug usage. Assume also that if a drug incident has occurred, Harry can detect it. Find
   (a) the probability that Harry detects drug usage on his first visit.
   (b) the probability that four drug usage incidents happen before Harry’s 5th visit.
   (c) the expected time till Harry finds evidence of drug usage.