Section 7

1. A storm begins and the old extension bridge over the precipitous ravine starts to wobble and shake. The storm will be long and the bridge is old and fraying, and its lifetime is exponentially distributed with mean 2 hours. One hour later, our hero Indiana Jones arrives at the bridge. In the storm, the time taken to cross the bridge is an exponential random variable with mean 30 minutes.

(a) What is the probability that Indiana Jones is able to cross the bridge before it collapses?
(b) What is the probability that Indiana will fall into the ravine?

2. A post office hires three clerks, each of which can serve customers in a time that is exponentially distributed with rate $\mu$. The clerks return from their coffee break and find fifty customers in line. Let $T$ be the time until all customers are served, assuming no further arrivals occur. Find $E(T)$.

3. Suppose $X$ is exponentially distributed with rate $\lambda$. Let $Y = \lfloor X \rfloor$, where $\lfloor x \rfloor$ is the largest integer that is less than or equal to $x$. Find the distribution of $Y$.

4. A continuous random variable is memoryless if for all $s, t \geq 0$, we have

$$P(X > s + t | X > s) = P(X > t).$$

A discrete random variable is memoryless if for nonnegative $s, t \geq 0$, we have

$$P(X > s + t | X \geq s) = P(X > t).$$

(a) Prove that the exponential random variable is the only continuous random variable with the memoryless property. (Hint: Use the fact that $f(x) = \lambda x$, for $\lambda \in \mathbb{R}$, is the only type of continuous function satisfying the property

$$f(s + t) = f(s) + f(t)$$

for any real numbers $s$ and $t$.)

(b) Find a memoryless discrete random variable.