1. A production process changes states \{1, 2, 3\} in accordance with a Markov Chain with transition probability matrix

\[
P = \begin{pmatrix}
0 & \frac{1}{3} & \frac{2}{3} \\
\frac{1}{3} & 0 & \frac{1}{3} \\
1 & 0 & 0
\end{pmatrix}.
\]

If the plant earns $20M, $30M and $40M whenever the state is 1, 2 and 3 respectively, what is long-run earning rate of the plant?

2. Harry has set up a booth at a local computer show and is taking orders. Harry is exceedingly slow and each order takes 3 minutes to fill. While each order is being filled there is probability \(p_j\) that \(j\) more customers will arrive and join the line. Assume \(p_0 = 0.2; p_1 = 0.2\) and \(p_2 = 0.6\). Harry cannot take a coffee break until a service is completed and no one is waiting in line to order more of his software. What is the probability that Harry will ever take a coffee break?

3. For the Markov chain with states 1, 2, 3 whose transition probability matrix \(P\) is as specified below find the mean time spent in each of the transient states, starting from any of the transient states 1, 2.

\[
P = \begin{pmatrix}
0.4 & 0.2 & 0.4 \\
0.1 & 0.5 & 0.4 \\
0 & 0 & 1
\end{pmatrix}
\]

4. Friday night is amateur night at Happy Harry’s Restaurant where a seemingly infinite stream of performers dreaming of stardom perform in lieu of the usual professional floor show. The quality of the performers falls into five categories with "1" being the best and "5" being unspeakably atrocious, representing for Harry’s discriminating clientele an exceedance of the threshold of pain which may cause a riot. The probability a class 5 performer will cause a riot is .3. After the riot is quelled, performances resume - the show must go on. Since performers tend to bring along friends of similar talent to perform, it is found that the succession of states on Friday night at Happy Harry’s can be modeled as a six-state Markov chain, where state 6 represents "riot" and state "\(i\)" represents a class "\(i\)" performer, \(1 \leq i \leq 5\). The transition matrix is given by

\[
P = \begin{pmatrix}
0.05 & 0.15 & 0.3 & 0.3 & 0.2 & 0 \\
0.05 & 0.3 & 0.3 & 0.3 & 0.05 & 0 \\
0.05 & 0.2 & 0.3 & 0.35 & 0.1 & 0 \\
0.05 & 0.2 & 0.3 & 0.35 & 0.1 & 0 \\
0.01 & 0.1 & 0.1 & 0.1 & 0.39 & 0.3 \\
0.2 & 0.2 & 0.2 & 0.2 & 0 & 0
\end{pmatrix}
\]
To play it safe Harry starts the evening off with a class 2 performer. What is the probability that a star is discovered (a class 1 performer) before a riot is encountered?

5. Suppose there is a machine shop with two independent machines and a repair person, and let \( \{X_n, n \geq 0\} \) be the number of machines running in the shop at time \( n \). Let

\[
\begin{pmatrix}
0 & 0.0009 & 0.0582 & 0.9409 \\
1 & 0.0006 & 0.0488 & 0.9506 \\
2 & 0.0004 & 0.0392 & 0.9604
\end{pmatrix}
\]

Given \( X_0 = 2 \), what is the expected time until both machines are down for the first time?

6. (Backup) A process moves on the integers \( S = \{0, 1, \ldots, N\} \). Starting from 1, and on each successive step, it moves to an integer greater than its present position, moving with equal probabilities to each of the remaining larger integers. State \( N \) is absorbing. Find the expected number of steps to reach state \( N \). (Hint: Use first step analysis.)