Problem 1  Compute the following limits, if they exist. If a limit does not exist, explain why. Make sure you justify the use of any rules or theorems.

(a) \( \lim_{\theta \to 0} \frac{1 - \cos(\theta)}{\sin(\theta)} \)

(b) \( \lim_{x \to 0} \frac{\sin(x)}{\cos(x)+x} \)

(c) \( \lim_{x \to 0} (\cos x)^{\frac{1}{x}} \)

(d) \( \lim_{x \to 0} \frac{\frac{x^3}{\sin 2x - 3x}}{x^3} \)

(e) \( \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{2\tan x} \)

(f) \( \lim_{x \to +\infty} (x^2 + x + 1)e^{-x} \)
**Problem 2**  Find all horizontal asymptotes of \( f(x) = \frac{2x + e^x}{3x + e^x} \)

**Problem 3**  Consider \( \lim_{x \to 0^+} \frac{x}{e^{1/x}} \)

(a) Try using L’Hospital’s rule to find the limit. Can you do it?

(b) Find the limit some other way.

**Problem 4**  Show that \( \lim_{k \to \infty} \left(1 + \frac{r}{k}\right)^k = e^r \)