Math 1110: In-Class Problems for 3.2

Problem 1  Determine whether the following functions are differentiable at \( x = 0 \). You must completely justify your answer.

(a)  \( B(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases} \)

(b)  \( C(x) = x \cdot |x| \)

Problem 2  Evaluate the following expression. You do not need to simplify your answers.

\( f'(x) \) if \( f(x) = \sin(\ln(5\pi)) \)
Problem 3  Determine whether the following statements are (always) true or (at least sometimes) false, and circle your response. Please give a brief explanation (in complete sentences!) - a reason why it’s true, or an example where it fails.

(a) If a differentiable function $h(x)$ can be written as a product $h(x) = f(x) \cdot g(x)$, then each factor $f(x)$ and $g(x)$ is also differentiable.

(b) If a function $f$ is continuous at $x = 0$, then $f$ is differentiable at $x = 0$.

(c) If a function $f$ is differentiable at $x = 0$, then $f$ is continuous at $x = 0$.

Problem 4  Find constants $a$ and $b$ such that the function

\[ f(x) = \begin{cases} 
-e^{\sin(x)} & \text{if } x < 0 \\
ax + b & \text{if } x \geq 0
\end{cases} \]

is differentiable at $x = 0$. 