Math 1110: Solutions for 3.1 In-Class Problems

Problem 1  Does the graph of \( y = x^{2/5} \) have a tangent line at \( x = 0 \)? If it does, find it. If it does not, explain why not. Tip: Read the introduction to problems 35 and 36 on page 125)

\[
\lim_{x \to 0} \frac{x^{2/5} - 0^{2/5}}{x - 0} = \lim_{x \to 0} \frac{1}{x^{3/5}}
\]

\[
\lim_{x \to 0^-} \frac{1}{x^{3/5}} = -\infty
\]

\[
\lim_{x \to 0^+} \frac{1}{x^{3/5}} = \infty
\]

Since this limit does not exist, there is not a tangent line at \( x = 0 \). With infinite limits, we would get a vertical tangent if they both were \( \infty \) or both were \( -\infty \). If you graph this function you will see that there is a sharp cusp at \( x = 0 \).

Problem 2  If \( f(t) = \sqrt{|4 - t|} \), what is \( f'(4) \)?

\[
\lim_{t \to 4} \frac{f(t) - f(4)}{t - 4} = \lim_{t \to 4} \frac{\sqrt{|4 - t|} - \sqrt{|4 - 4|}}{t - 4}
\]

\[
= \lim_{t \to 4} \frac{\sqrt{|4 - t|}}{t - 4}
\]

\[
\lim_{t \to 4^+} \frac{\sqrt{|4 - t|}}{t - 4} = \lim_{t \to 4^+} \frac{\sqrt{(4 - t)}}{t - 4}
\]

\[
= \lim_{t \to 4^+} \frac{\sqrt{4 - t}}{t - 4} = \infty
\]

\[
\lim_{t \to 4^-} \frac{\sqrt{|4 - t|}}{t - 4} = \lim_{t \to 4^-} \frac{\sqrt{4 - t}}{t - 4}
\]

\[
= \lim_{t \to 4^-} \frac{\sqrt{4 - t}}{(4 - t)}
\]

\[
= \lim_{t \to 4^-} \frac{\sqrt{4 - t}}{-\sqrt{4 - t}} = -\infty
\]

\( f'(4) \) does not exist since the two one-sided limits are not equal.
Problem 3  What is the instantaneous rate of change of \( y = \frac{1}{x-1} \) at \( x = 3 \)?

The instantaneous rate of change is

\[
\lim_{x \to 3} \frac{1}{x-3} = \lim_{x \to 3} \frac{1}{x-3} - \frac{1}{x-3} = \lim_{x \to 3} \frac{2}{x-3} - \frac{x-1}{x-3} = \lim_{x \to 3} \frac{2 - x + 1}{2(x-1)(x-3)}
\]

\[
\lim_{x \to 3} \frac{3 - x}{-2(x-1)(3-x)} = \lim_{x \to 3} \frac{1}{-2(x-1)} = \frac{1}{-2(3-1)} = -\frac{1}{4}
\]

Problem 4  What is the equation for the tangent to the graph of \( y = \sin x \) at \( x = 0 \)?

The slope of the tangent line is

\[
m = \lim_{x \to 0} \frac{\sin x - \sin 0}{x - 0} = \lim_{x \to 0} \frac{\sin x}{x} = 1
\]

We have the slope so now we need to find the point on the graph at \( x = 0, \ y = \sin 0 = 0 \). Then the tangent line is the line with a slope of 1. It is \( y - 0 = 1(x - 0) \), i.e. the line \( y = x \).

Problem 5  Find the equation of the straight line having slope \( \frac{1}{4} \) that is tangent to the curve \( y = \sqrt{x} \).

For this problem, we need to find the value if \( z \) that makes

\[
\lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h} = \frac{1}{4}
\]

\[
\lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h} \left( \frac{\sqrt{x + h} + \sqrt{x}}{\sqrt{x + h} + \sqrt{x}} \right)
\]

\[
= \lim_{h \to 0} \frac{(x + h) - (x)}{h(\sqrt{x + h} + \sqrt{x})} = \lim_{h \to 0} \frac{h}{h(\sqrt{x + h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x + h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}
\]

This equals \( 1/4 \) when \( x = 4 \), so the point that we want to find the tangent line at is \( (4, \sqrt{4}) = (4, 2) \). The desired line is then

\[
y - 2 = \frac{1}{4}(x - 4)
\]

\[
y = \frac{x}{4} + 1
\]