Problem 1 Twins Problem

There is a bit of trivia about the author of your textbook, Dr. George Thomas, that very few people know. He has an evil twin sister named Shasta. Although he loves his sister dearly, she disliked him, and tries to be different from him in all things.

Last winter, they both went on vacation. Dr. Thomas went to Hawaii. Shasta had planned on going to Aruba, but she decided against it. She hates her brother so much that she was afraid there would be a chance that they might experience the same temperature at the same time, and that prospect was distasteful to her. So she decided to vacation in northern Alaska.

After a few days, Dr. Thomas received a call: “This is Shasta. I am very cold and uncomfortable here. That’s good, since you are undoubtedly warm and comfortable, and I want us to be different. But I’m not sure why I should be the one in northern Alaska. I think we should switch places for the last half of our trip.”

"It is only fair" he agreed.

So they each traveled again. Dr. Thomas took a trip from Hawaii to Alaska, while Shasta took a trip from Alaska to Hawaii. They each traveled their own different routes, perhaps stopping at different places along the way. Eventually, they had reversed locations. Dr. Thomas was shivering in Alaska, Shasta was in Hawaii, warm and happy. She received a call from her brother.

"Hi Stasta. Guess what? At some time during our travels, we were experiencing exactly the same temperature at the same time. So Ha!"

Is Dr. Thomas right? Has Good triumphed over evil? He would try to write out a proof of his statement, but his hands are too frozen to grasp his pen. Help him out. Either prove him right, or prove him wrong, using mathematics.

Problem 2 Determine whether the following statement is always, sometimes or never true. Briefly justify your answers using complete sentences. As appropriate, provide equations and graphs, and reference theorems, definitions, and properties to support your answers.

- If a statement is true, provide a reason why it is true.
- If a statement is false, provide an example in which it fails.
- If a statement is sometimes true and sometimes false, provide an example that shows when it is true and an example that shows when it is false.

Let $f$ be a continuous function on $[-1,1]$ such that $f(1) = -f(-1)$.

(a) $f(0) = 0$

(b) For some $x$ with $-1 \leq x \leq 1$, $f(x) = 0$

(c) $f(x) = -1$ for all $x < 0$ and $f(x) = 1$ for all $x > 0$
Problem 3  Swimming to Shore

A swimmer crosses a river starting at point $A$ and ending at point $B$, following the path shown below. Prove that for some value $x$, the swimmer’s distance $d(P_x, A)$ from $A$ is the same as the distance $d(P_x, B)$ from $B$. 

![Swimmer's Path Diagram]