\[ y = e^{\cos 4x} \]

Find \( \frac{dy}{dx} \) using implicit differentiation.

3) A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?
1) \( y \sin y = e^y \cos 4x \)

Find \( \frac{dy}{dx} \) using implicit differentiation.

\[
\frac{d}{dx} \left( y \sin y \right) = e^y \cdot \sin 4x \cdot 4 + e^y \cdot \cos 4x \cdot \frac{dy}{dx}
\]

\[
8 \cos 4x \cdot \frac{dy}{dx} + y \sin y = 4e^y \sin 4x + e^y \cos 4x \cdot \frac{dy}{dx}
\]

\[
\frac{dy}{dx} = \frac{4e^y \sin 4x - y \sin y}{8 \cos 4x - 4e^y \cos 4x}
\]

\[
\frac{dy}{dx} = \frac{y(e^y \sin 4x + \sin y)}{e^y \cos 4x - 8 \cos 4x}
\]

2) Find the derivative of \( y^2 \sin(\sqrt{2})^x \ln x \)

\[
\frac{dy}{dx} = e^{\ln x} \cdot \ln(\sin(\sqrt{2})) \cdot \left( \ln x \cdot \frac{1}{\sin(\sqrt{2})} \cdot \cos(\sqrt{2}) \cdot \frac{1}{x} + \frac{1}{x} \cdot \ln(\sin(\sqrt{2})) \right)
\]

\[
= e^{\ln x} \ln(\sin(\sqrt{2})) \cdot \left( \ln x \cdot \frac{\cos(\sqrt{2}) + \ln(\sin(\sqrt{2}))}{\sin(\sqrt{2})} \right)
\]

3) A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

\[
A: \quad \text{height} = 300 \text{ ft}
\]

\[
x = \text{distance of kite from girl} \quad \frac{dx}{dt} = 25 \text{ ft/sec}
\]

\[
c = \text{string length of kite}
\]

Looking for \( \frac{dc}{dt} \) (how fast the next let out string)

When \( c = 500 \text{ ft} \)

\[
x^2 + 300^2 = c^2
\]

\[
x = 400 \text{ ft}
\]

\[
\frac{dx}{dt} = \frac{400}{500} \cdot (25)
\]

\[
\frac{dc}{dt} = 20 \text{ ft/sec}
\]
1. Show that the point $(4, 2)$ lies on the curve $x^3 + y^3 - 9xy = 0$. Then find the tangent and normal line to the curve.

2. Find the derivative of the function.

$$y = x \tan(2 \sqrt{5} x) + 7$$

3. $s = 4t^2 - 3t + 6 \quad 1 \leq t \leq 5m$
   a. Find the displacement at the given time interval.
   b. Find the speed and acceleration at the endpoints.
Answer Key

1. \(x^3 + y^3 - 9xy = 0\)  \((4, 2)\)
   \(4^3 + 2^3 - 9(4)(2) = 0\)  \((4, 2)\) lies on the curve.

\[
3x^2 + 3y^2 \frac{dy}{dx} - 9y - 9x \frac{dy}{dx} = 0
\]
\[
(3y^2 - 9x) \frac{dy}{dx} = 9y - 3x^2
\]
\[
\frac{dy}{dx} = \frac{9y - 3x^2}{3y - 9x} = \frac{3y - x^2}{y - 3x}
\]

Slope at \((4, 2)\)
\[
= \frac{3(2) - 4^2}{2^2 - 12} = \frac{-10}{-8} = \frac{5}{4}
\]

Tangent line
\[
y - 2 = \frac{5}{4}(x - 4)
\]
\[
y = \frac{5}{4}x - 3
\]

Normal line
\[
y - 2 = -\frac{4}{5}(x - 4)
\]
\[
y = -\frac{4}{5}x + \frac{16}{5} + \frac{10}{5}
\]
\[
y = -\frac{4}{5}x + \frac{26}{5}
\]

2. \(y = x \tan(2\sqrt{3}x) + 7\)
   \(y' = \tan(2\sqrt{3}x) + x \sec^2(2\sqrt{3}x) \cdot x^{-1/2}\)
   \(= \tan(2\sqrt{3}x) + \sqrt{x} \sec^2(2\sqrt{3}x)\)

3.
   a. \(S(5) - S(1) = (4(25) - 3(1) + 6) - (4 - 3 + 6)\)
      \(= (100 - 15 + 6) - 7 = 91 - 7 = 84\) m
   b. \(\frac{ds}{dt} = 8t - 3 = v\)
      \(V(1) = 5\) m/sec  \(V(5) = 37\) m/sec
      \(\frac{dv}{dt} = 8\) m/sec^2
Questions for Prelim

1. Find equations for the lines that are tangent and normal to the curve at the given point.

\[ x + \sqrt{y} = 6 \quad (4,1) \]
\[ \frac{dy}{dx} = \frac{-5}{4} \]
\[ dx \]
\[ \tan \text{gent line: } y = 1 - \frac{5}{4}(x-4) \]
\[ y = \sqrt{5}(x-4)+1 \]

2. A particle moves along the curve \( y = x^{3/2} \) in the first quadrant in such a way that its distance from the origin increases at the rate of 11 units per second. Find \( \frac{dx}{dt} \) when \( x = 3 \).

\[ D^2 = x^2 + y^2 \]
\[ D^2 = 9 + 27 \]
\[ D = 6 \]
\[ \frac{dx}{dt} = \frac{\partial x}{\partial (6, 11)} \]
\[ \frac{dx}{dt} = \frac{4 \text{ units/sec}}{6 + \sqrt{7}} \]

3. Use logarithmic differentiation to find the derivative of \( y \) with respect to the appropriate variable.

\[ y = 2(x^2 + 1) \sqrt{x} \cos^2 x \]
\[ \frac{dy}{dx} = \frac{3x(x^2 + 1)}{2(x^2 + 1)x^2} \]
\[ = \frac{3x}{2x} \]
\[ = \frac{3}{2} \]
\[ \frac{dy}{dx} = \frac{1}{x} \left( \frac{2x}{x^2 + 1} \right) - \frac{1}{x} \left( \frac{1 - x^2}{2} \right) \]
\[ \frac{dy}{dx} = y \left( \frac{x}{x^2 + 1} + \sin 2x \right) \]
1) A spotlight on the ground shines on a wall 50 m away. If a man 2 m tall walks from the spotlight towards the building at a speed of 1/2 m/s, how fast is his shadow decreasing when he is 4 m from the building?

\[ \frac{dx}{dt} = \frac{1}{2} \text{ m/s} = \text{man walks toward building} \]

Find \( dh/dt \) (rate at which the man's shadow decreases)

Similar triangles, therefore sides are proportional:
\( \Delta abc \) & \( \Delta ade \) are similar

\[
\frac{x}{h} = \frac{2}{50} \quad \Rightarrow \quad \frac{xh}{50} = \frac{1}{25} \\
\frac{xh}{100} = \frac{xh}{100} = \frac{4m(h)}{100} \\
\frac{dx}{dt} \cdot (xh) = 100 \\
\frac{dx}{dt} \cdot h \frac{dh}{dt} + h \frac{dx}{dt} = 0 \\
\frac{dh}{dt} = -\frac{h \frac{dx}{dt}}{x} \\
\text{when he man is 4 m from building, } x = 46m \\
\frac{dh}{dt} = -\frac{25}{46} \text{ m/s} \\
\begin{bmatrix} 4m + x = 50m \quad \rightarrow \quad x = 46m \end{bmatrix}
\]

2) Find an equation of the line tangent to the graph of \( y = \ln(x^2 + 4) - x + \tan^{-1}(\pi/2) \) at \( x = 2 \)

\[
y' = \frac{x^2 + 4}{x^2 + 4} - \left\{ \frac{1}{2 + x^2} \cdot 2 + \tan^{-1}(\pi/2) \right\} \\
y = \frac{2x}{x^2 + 4} \cdot \left\{ \frac{x}{2 + x^2} + \tan^{-1}(\pi/2) \right\} \\
y' = \frac{2x}{x^2 + 4} - \frac{1}{2 + x^2} + \tan^{-1}(\pi/2) \]
\[ y' = \frac{2x}{x^2 + 4} - \left\{ \frac{2x}{4 + x^2} + \tan^{-1}\left( \frac{x}{2} \right) \right\} \]

\[ y' = \frac{2x}{x^2 + 4} - \frac{2x}{4 + x^2} - \tan^{-1}\left( \frac{x}{2} \right) \]

\[ y' = -\tan^{-1}\left( \frac{x}{2} \right) \]

slope = \[ y' = \tan^{-1}\left( \frac{x}{2} \right) = \tan^{-1}\left( 1 \right) = -\frac{\pi}{4} \]

\[ y = \ln\left( \frac{x^2 + 4}{x} - \tan^{-1}\left( \frac{x}{2} \right) \right) \]

\[ y = \ln\left( \frac{2^2 + 4}{x} - 2 + \tan^{-1}\left( \frac{x}{2} \right) \right) \]

\[ y = \ln\left( \frac{8}{x} - \frac{\pi}{2} \right) \]

\[ y = \ln\left( 8 - \frac{\pi}{2} \right) = -\frac{\pi}{4} (x - 2) \]

\[ y = -\frac{\pi}{4} x + \ln 8 \]

\[ \boxed{\text{3) Find an equation of the line tangent to the graph of } (x^2 + y^2) = 8 x^2 y^2 \text{ at point } (-1, 1)} \]

\[ \frac{d}{dx} (x^2 + y^2)^3 = 3 (x^2 + y^2)^2 \cdot (2x + 2y \cdot \frac{dy}{dx}) = 8 x^2 y \frac{dy}{dx} + y^2 \]

\[ 6 (x^2 + y^2)^2 \cdot 6 y \frac{dy}{dx} (x^2 + y^2)^2 = 16 x^2 y \frac{dy}{dx} + y^2 \]

\[ \frac{dy}{dx} \left[ 6y (x^2 + y^2)^2 - 16x^2 y^2 \right] = 16 x y^2 - 6(x^2 + y^2)^2 \]

\[ \frac{dy}{dx} = \frac{16 x y^2 - 6(x^2 + y^2)^2}{6y (x^2 + y^2)^2 - 16x^2 y} \]

\[ \text{slope at } (-1, 1) = m = y' = \frac{16 (-1)(-1)^2 - 6(-1)(1)^2}{6(1) [(1)^2 + (1)^2] - 16 (-1)(1)} = \frac{8}{8} = 1 \]

\[ = 1 \]

\[ \text{Equation of tangent line: } \]

\[ y - (1) = 1(x - 1) \]

\[ \boxed{y = x + 2} \]
1. A ladder 5 meters long is leaning against a vertical wall. The base of the latter is moving away from the wall at a rate of 1 meter per second. A man is climbing up the ladder at a rate of 1 meter per second. Find the vertical velocity of the man when he is 1 meter from the bottom of the ladder, and the base of the ladder is 3 meters away from the wall.

2. Find \( \frac{d}{dx} \log_2(y^2+3y)=x^3+\cos x \)

3. Derive \((x^x)(\sin x)\)

4. A cylinder of water with a fixed volume is decreasing in radius at a rate of 1 cm/s. Assuming radius \( r \) cm and height \( h \) cm, at what rate is the height increasing?
\[
\log_2 \left( y^2 + 3y \right) = x^3 + \cos x
\]
\[
\frac{1}{(y^2 + 3y) \ln 2} \cdot \left( 2y \frac{dy}{dx} + 3 \frac{dy}{dx} \right) = 3x^2 - \sin x
\]
\[
(2y + 3) \frac{dy}{dx} = \left( 3x^2 - \sin x \right)(y^2 + 3y)(\ln 2)
\]
\[
\frac{dy}{dx} = \frac{(3x^2 - \sin x)(y^2 + 3y)(\ln 2)}{2y + 3}
\]

\[
\frac{d}{dx} x^x \sin x
\]

\[
y = x^x \sin x
\]
\[
\ln y = \ln (x^x \sin x)
\]
\[
\ln y = \ln x^x + \ln \sin x
\]
\[
\ln y = x \ln x + \ln \sin x
\]
\[
\frac{dy}{dx} = x \left( \frac{1}{x} \right) + (1) \ln x + \frac{1}{\sin x} \cos x
\]
\[
\frac{dy}{dx} = 1 + \ln x + \cot x
\]
\[
\frac{dy}{dx} = x^x \sin x \left( 1 + \ln x + \cot x \right)
\]
\[
\frac{d}{dx} \left( x^x \sin x \right) = x^x \sin x \left( 1 + \ln x + \cot x \right)
\]

---

4

\[
V = \pi r^2 h
\]
\[
\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi r \frac{dr}{dt} h
\]
\[
0 = \frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi r \frac{dr}{dt} h
\]
\[
-2\pi r \frac{dr}{dt} h = \frac{dV}{dt} - \pi r^2 \frac{dh}{dt}
\]
\[
\frac{-2\pi r \frac{dr}{dt} h}{\pi r^2} = \frac{dV}{dt} - \frac{dh}{dt}
\]
\[
\frac{2h}{r} = \frac{dh}{dt}
\]