Math 1110: 3.4 Solutions

Problem 1

velocity of a particle:
\[ s'(t) = v(t) = 3t^4 - 20t^3 + 17t + \tan^{-1}(20). \]
acceleration, \( a(t) = v'(t) = s''(t) \)
\[ v'(t) = a(t) = 12t^3 - 60t^2 + 17 \]

Problem 2

velocity vs time:

![Graph of velocity vs time]

a) acceleration vs time:

![Graph of acceleration vs time]

acceleration is the change in velocity over time, or the derivative.

speed vs time:

![Graph of speed vs time]

speed doesn't depend on direction like velocity, so it can't be negative - it's the absolute value of velocity.
Problem 2 (cont)

b) the particle is at rest at \( t = 2 \) and \( t = 8 \) seconds, where speed and velocity are 0.
c) the particle’s speed is increasing over \((2, 5)\) and \((8, 10)\), since the absolute value of velocity (speed) is increasing over these intervals (the slope of the speed graph is positive).

Problem 3

One approach to this problem would be to look at each graph individually, to predict what its derivatives would look like.

\[\text{a)}\]

![Graph a)

Its derivative would be:
positive, zero, negative, zero, then positive.

So the graph would look something like:

![Graph a) derivative]

\[\text{b)}\]

![Graph b)

Its derivative would be:
close to zero, increase, zero, negative (reach a minimum), then decrease, zero, then increase (reach a minimum), then decrease, zero, then increase towards zero.

It would look something like:

![Graph b) derivative]

Nothing looks like this, so we can assume this is not \( f(x) \) or \( f'(x) \).

Fits with a) since none of the graphs look like a’s derivatives, we can say that:

\[ f(x) = c \]
\[ f'(x) = b \]
\[ f''(x) = a \]

Double checking by looking at the graph of a) would show that this is indeed right.

Problem 4 (3.4#13) & Problem 5 (3.4#26)

See homework solutions.