1. \( f(x) = x^{\frac{1}{3}} \)
   \[ f'(x) = \frac{1}{3} x^{-\frac{2}{3}} \]
   a) The linearization at \( x = a = 8 \) is
   \[ L(x) = f(8) + f'(8)(x-8) \]
   \[ = 8^{\frac{1}{3}} + \frac{1}{3 \cdot 8^{\frac{2}{3}}} (x-8) \]
   \[ = 2 + \frac{1}{12} (x-8) \]
   b) Use \( L(x) \) to approximate \( 3\sqrt{8.24} \)
   \[ 3\sqrt{8.24} \approx L(8.24) = 2 + \frac{1}{12} (8.24-8) \]
   \[ = 2 + \frac{0.24}{12} = 2 + 0.02 = 2.02 \]
   [Side note - a computer calculates \( 3\sqrt{8.24} = 2.0198 \)]

2. Find the linearization of \( f(x) = (1+x)^{2011} \) at \( x = 0 \)
   a) \( f'(x) = 2011 (1+x)^{2010} \)
   \( L(x) = f(0) + f'(0)(x-0) \)
   \[ = (1+0)^{2011} + 2011 (1+0)^{2010} (x) \]
   \[ = 1 + 2011 x \]
   b) Estimate \( f\left(\frac{1}{2011}\right) \approx L\left(\frac{1}{2011}\right) = 1 + 2011 \left(\frac{1}{2011}\right) = 2 \).
3) Estimate \( \tan^{-1}(2) \) by linear approximation.

a) Let \( g(x) = \tan^{-1}(x) \). Find the linearization at \( x = 1 \).

\[
L(x) = g(1) + g'(1)(x - 1) = \tan^{-1}(1) + \frac{1}{1 + 1^2} (x - 1)
\]

\[
= \frac{\pi}{4} + \frac{x}{2} \left( x - 1 \right)
\]

\[
= \frac{\pi}{4} + \frac{1}{2} (x - 1)
\]

b) \( g(2) \approx L(2) = \frac{\pi}{4} + \frac{1}{2} (2 - 1) = \frac{\pi}{4} + \frac{1}{2} \)

c) When \( x \) is positive (like it is at \( x = 1 \) and \( x = 2 \)), \( g''(x) \) is negative. This means that \( g'(x) \) is decreasing and that the estimate will be an overestimate.

If we calculate it, \( \tan^{-1}(2) = 1.107148 \) and \( L(2) = \frac{\pi}{4} + \frac{1}{2} = 1.285398 \).
Let
- \( h \) = height of pole
- \( d \) = distance from pole to tip of shadow
- \( x \) = distance from pole to man
- \( l \) = length of man's shadow
- \( m \) = height of the man.

We know that \( h, m \) are constants also by similar triangles.

\[
\frac{m}{l} = \frac{h}{d}
\]
and \( x + l = d \)

We want to relate \( \frac{dx}{dt} \) to \( \frac{dl}{dt} \).

\[
\frac{m}{l} = \frac{h}{x + l}
\]

\[
m(x + l) = hl
\]

\[
\frac{d}{dt} m(x + l) = \frac{d}{dt} hl
\]

\[
m \left( \frac{dx}{dt} + \frac{dl}{dt} \right) = h \frac{dl}{dt}
\]

\[
m \frac{dx}{dt} = h \frac{dl}{dt} - m \frac{dl}{dt} = (h - m) \frac{dl}{dt}
\]

\[
\frac{dx}{dt} = \frac{h - m}{m} \frac{dl}{dt}
\]

constant.

So the answer is \( \boxed{\text{They are a constant multiple of the other}} \).
\[ H = \text{height of shadow on wall} \]
\[ h = \text{height of woman} \]
\[ x = \text{distance of the woman from the light} \]
\[ d = \text{distance from the light to the wall}. \]

Know: \( h \) and \( d \) are constant.

We want to relate \( \frac{dx}{dt} \) to \( \frac{dH}{dt} \).

By similar triangles we know that
\[ \frac{h}{x} = \frac{H}{d} \rightarrow \frac{dh}{dt} = \frac{xH}{d}. \]

Or \[ dH = \frac{dh}{x} \]

\[
\frac{d}{dt} H = \frac{d}{dt} \frac{dh}{x} \]

\[
\frac{dH}{dt} = dh \left( -\frac{1}{x^2} \right) \frac{dx}{dt} = -\frac{dh}{x^2} \frac{dx}{dt}. \]

So the answer is (5): It depends on how close the woman is to the light (or wall).