We will use Wednesday 12/02 and Friday 12/04 for a comprehensive review of the entire course, in preparation for the final. As with previous review times, I’ve compiled a list of interesting questions¹ that are representative of the entire course. However, this list is not authoritative, and comes with no guarantees of similarity to the final.

1. Consider the function defined by $F(x) = \int_0^x \sqrt{3 + \sin t} \, dt$. Without trying to find an explicit formula for $F$,
   (a) Show that $F(1) \leq 2$
   (b) Determine whether $F$ is concave up or concave down on $0 < x < 1$.
   (c) Find $\int_1^2 \sqrt{3 + \sin t} \, dt$ in terms of values of $F$.

2. Suppose that $f'(x) = e^{x^2}$, and $f(0) = 10$. One can conclude from the mean value theorem that $A < f(1) < B$ for which two numbers $A$ and $B$?

3. How should the parameter $\lambda$ be chose so that $f(x) = e^{\lambda x}/(1 + 2\sin x)$ remains as close to 1 as possible, when $x \approx 0$? Use this value of $\lambda$ to estimate $f(0.1)$ to two decimal places.

4. If $\int_0^x f(t)dt = e^{2x} \cos x + c$, find the value of the constant $c$ and the function $f(t)$.

5. Suppose that $f(x + y) = f(x)f(y)$ and $f(x) = 1 + xg(x)$, where $\lim_{x \to 1} g(x) = 1$. Show that the derivative $f'(x)$ exists for all $x$, and that $f'(x) = f(x)$.

6. For the function $f(x) = 3x^5 - 5x^3 + 1$, sketch the graph over a suitable interval showing all of the local maxima, local minima, and points of inflection. Also show the approximate location of all roots, in the form $[n, n + 1]$, where $n$ is an integer. (That is, you don’t have the find the roots exactly, but you do need to show how you know where they are.)

7. For the function $f(x) =$ \begin{cases} ax + b & x < 0 \\ 1 - x + x^2 & x \geq 0 \end{cases}
   (a) Find all $a$ and $b$ or which $f$ will be continuous.
   (b) Find all $a$ and $b$ or which $f$ will be differentiable.

¹Citation: most of these questions have been selected from MIT OpenCourseWare sample exams.
8. Find the area between \( y = 8 \cos x \) and \( y = \sec^2 x \), for \( x \in \left[ -\frac{\pi}{3}, \frac{\pi}{3} \right] \).

9. The function \( F(x) = \int_0^x t^2 e^{-t^2} \, dt \) is not elementary, but it arises when computing the standard deviation of a normal distribution.
   
   (a) Express \( \int_0^9 \sqrt{u} e^{-u} \, du \) in terms of values of \( F \).

   (b) Estimate \( F \) by showing that \( F(x) \leq x^3/3 \), for \( x > 0 \).

10. Solve the initial value problems:
   
   (a) \( \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \), \( y(0) = 0 \)

   (b) \( \frac{dy}{dx} = \frac{1}{1+x^2} - 1 \), \( y(0) = 1 \)

11. Evaluate the following limits:
   
   (a) \( \lim_{u \to 0} \frac{\tan 2u}{u} \)

   (b) \( \lim_{h \to 0} \frac{e^h - 1}{h} \)

   (c) \( \lim_{x \to \infty} \frac{\sqrt{x+5}}{\sqrt{x+5}} \)

12. The bottom of the legs of a three-legged table are the vertices of an isosceles triangle with sides lengths 5,5,6. The legs are to be braced at the bottom by three wires in the shape of a Y. What is the minimum length of wire needed?

13. For the curve given by \( x^2 y + y^3 + x^2 = 8 \), find all points \((x, y)\) where the tangent line to the curve is horizontal.

14. Derive the formula for \( \frac{d}{dx} \sin^{-1} x \) using implicit differentiation. Assume the standard domain \( \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \) for \( \sin^{-1} x \).