Multivariate normals are distributions over vectors. Gaussian processes are distributions over functions $f : \mathbb{R}^d \rightarrow \mathbb{R}$ is the kernel function $l \sim GP(\mu, k)$

- $\mu : \mathbb{R}^d \rightarrow \mathbb{R}$ is the mean field
- $k : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}$ is the covariance function

Objective is to estimate, for a symmetric positive definite matrix $K = \mathbb{K}^{1/2} \mathbb{L} \mathbb{L}^{T} \mathbb{K}^{1/2}$

- $\mathbb{L}$ is the lower triangular Cholesky factor
- $K_{x,x}$ is the covariance matrix

Log det accuracy

- The data is 1000 points drawn from $N(0, 2)$
- The exact values are $(\bullet)$, Lanczos with diagonal replacement is $(\longrightarrow)$, Chebyshev with diagonal replacement is $(\cdots)$, Lanczos without diagonal replacement is $(\cdots)$, Chebyshev without diagonal replacement is $(\cdots)$

- Scaled eigenvalues is $(\times)$

Daily precipitation

- Precipitation data collected from $\approx 5500$ US weather stations
- Use induced grid of size $100 \times 100 \times 300$
- Use a subset of 12,000 entries for training with the exact method

Hickory Data Set

- Fitted log-Gaussian Cox process model to hickory tree counts in Michigan
- Area discretized using a 60 x 60 grid

- The scaled eigenvalue method was used in conjunction with the Fiedler bound

Discussion

- New method to efficiently compute the log det and derivatives
- Lanczos outperforms Chebyshev in general
- Our method is flexible and requires only fast MVMs
- Can explore same ideas for computing higher-order derivatives
- Supports diagonal correction and non-Gaussian likelihoods
- Implementations are available at:
  - https://github.com/dm65/GPML_SLD
  - https://github.com/jrg365/gpytorch