1. Find a parametrization of the line which is the intersection of the two planes $3x + 4y + z + 2 = 0$ and $3x + 4y + 4z + 5 = 0$.

Answer: Setting the two equations equal to each other yields $z = -1$. Substituting $z = -1$ into either equation, one obtains $3x + 4y + 1 = 0$. Solving for $y$ in terms of $x$, we get a parametrization for the line: $(x, -(1 + 3x)/4, -1)$.

2. How can you use dot products to tell whether two unit vectors are parallel, antiparallel, or neither?

Answer: The dot product of two such vectors is the cosine of the angle between them. If they are parallel their dot product equals 1, if they are antiparallel their dot product equals -1, and otherwise their dot product is strictly between -1 and 1.

3. Say $u = (1, 0)$, $v$ is unknown, but we are given that $u \cdot v = 1$ and the angle between them is $\theta = \pi/4$ (so $\cos \theta = \sqrt{2}/2$). How many solutions for $v$ are there? Find them.

Answer: Since $u \cdot v = 1$, $v = (1, 1)$ or $v = (1, -1)$.

4. Find the squared length of $u + v$, where $u, v \in \mathbb{R}^3$, in terms of dot products.

Answer: By definition, $\|u + v\|^2 = (u + v) \cdot (u + v) = u \cdot u + 2u \cdot v + v \cdot v$.

5. Find the projections of a point $(a, b)$ in $\mathbb{R}^2$ onto the $x$ axis and the $y$ axis. Check that this is consistent with the general formula for the projection of a vector $u$ along a vector $v$: $u_{\parallel} = \left(\frac{u \cdot v}{v \cdot v}\right)v$.

Answer: Clearly $(a, b)$ can be written as $(a, 0) + (0, b)$, the sum of a vector parallel to the $x$ axis and a vector parallel to the $y$ axis. To check the formula, let’s take any vector parallel to the $x$ axis as $v$, say, let $v = (2, 0)$. Then the projection of $(a, b)$ onto $v$ is $\left(\frac{2a}{2}\right)(-2, 0) = (a, 0)$. Obviously it would be more convenient to choose $v = (1, 0)$, but my point is that the formula works for any $v$ on the $x$ axis. The case of the projection onto the $y$ axis is the same.

6. Write the vector $(1, 3)$ as the sum of two vectors, one which is parallel to $(1, 1)$, and one which is perpendicular to $(1, 1)$. Draw a picture of these vectors to better see what’s happening.

Answer: The projection of $(1, 3)$ onto $(1, 1)$ is $\left(\frac{1}{2}\right)(1, 1) = (2, 2)$. Whatever is left over from $(1, 3)$ is the part perpendicular to $(1, 1)$, i.e., $(1, 3) - (2, 2) = (-1, 1)$ (note that $(-1, 1)$ is indeed perpendicular to $(1, 1)$).

7. Let’s say we have points $P, Q, R$ in $\mathbb{R}^2$. Show that the angle between $PQ$ and $PR$ is the same as the angle between $QP$ and $RP$. Draw pictures to explain what’s going on.
Answer: Recall that \( PQ = Q - P \), \( QP = P - Q = -PQ \), and similarly \( RP = -PR \). The cosine of the angle between \( PQ \) and \( PR \) is 
\[
\frac{PQ \cdot PR}{\|PQ\|\|PR\|} = \frac{(-1)^2QP \cdot RP}{\|QP\|\|RP\|} = \frac{QP \cdot RP}{\|QP\|\|RP\|}
\]
which is the cosine of the angle between \( QP \) and \( RP \).