1. Let \( f(x, y, z) = x + yz \), and let \( C \) be the line segment from \( P = (0, 0, 0) \) to \( Q = (6, 2, 2) \).

   a) Calculate \( f(c(t)) \) and \( ds = \|c'(t)\| \, dt \) for the parametrization \( c(t) = (6t, 2t, 2t) \) for \( 0 \leq t \leq 1 \).

   b) Evaluate \( \int_C f(x, y, z) \, ds \).

   [Notice that what you’re computing is the area under the curve \( f(c(t)) \|c'(t)\| \) for the interval \( 0 \leq t \leq 1 \).]

   Answer: a) \( c'(t) = (6, 2, 2) \), \( \|c'(t)\| = 2\sqrt{11} \) and \( f(c(t)) = 6t + 4t^2 \), so b) \[
   \int_C f(x, y, z) \, ds = \int_0^1 f(c(t)) \|c'(t)\| \, dt \\
   = 2\sqrt{11} \int_0^1 (6t + 4t^2) \, dt \\
   = \sqrt{11} \left( 6 + \frac{8}{3} \right).
   \]

2. Compute \( \int_C f \, ds \) where \( f(x, y) = y^3/x^7 \) and \( C \) is parametrized by \( y = x^4/4, 1 \leq x \leq 2 \).

   Answer: \( c(x) = (x, x^4/4), 1 \leq x \leq 2 \) is the familiar notation for the parametrization for \( C \). Then \( f(c(x)) = x^5/4^3 \), \( c'(x) = (1, x^3) \), and \( \|c'(x)\| = \sqrt{1 + x^6} \), so

   \[
   \int_C f \, ds = \frac{1}{4^3} \int_1^2 x^5\sqrt{1 + x^6} \, dx \\
   = \frac{1}{4^3} \int_2^{6_5} u^{1/2} \, du \\
   = \frac{1}{4^3} \frac{2}{6_3} \left[ u^{3/2} \right]_2^{6_5} \\
   = \frac{1}{4^3} \frac{1}{9} (6_5^{3/2} - 2^{3/2}).
   \]

3. Suppose that \( C \) has length 5. What is the value of \( \int_C \mathbf{F} \cdot d\mathbf{s} \) if:

   a) \( \mathbf{F}(P) \) is normal to \( C \) at all points \( P \) on \( C \)? Draw a cartoon of the curve \( C \), a point \( P \) on \( C \), the vector \( d\mathbf{s} \) at \( P \), and the vector \( \mathbf{F}(P) \) to help you reason.

   b) \( \mathbf{F}(P) \) is a unit vector pointing in the negative direction along the curve? Draw a cartoon to help you reason.

   Answer: See 1 Nov solutions.

4. Let \( \mathbf{F}(x, y) = (y^2, x^2) \), and let \( C \) be the curve \( y = x^{-1} \) for \( 1 \leq x \leq 2 \), oriented from left to right.
a) Calculate $\mathbf{F}(\mathbf{c}(t))$ and $d\mathbf{s} = \mathbf{c}'(t) \, dt$ for the parameterization of $\mathcal{C}$ given by $\mathbf{c}(t) = (t, t^{-1})$.

b) Calculate the dot product $\mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t) \, dt$ and evaluate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{s}$.

**Answer:**
a) $\mathbf{F}(\mathbf{c}(t)) = (t, t^2)$, $d\mathbf{s} = \mathbf{c}'(t) \, dt = (1, -t^{-2}) \, dt$, so b) $\mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t) \, dt = (t^{-2} - 1) \, dt$ and $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{s} = \int_{1}^{2} (t^{-2} - 1) \, dt = -1/2$.

5. Compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{s}$ for $\mathbf{F}(x, y) = \langle e^{y-x}, e^{2x} \rangle$ and $\mathcal{C}$ the piecewise linear path from $(1,1)$ to $(2,2)$ to $(0,2)$. [You should probably draw a picture of the path to avoid careless mistakes.]

**Answer:** See 1 Nov solutions.