1. Use spherical coordinates to compute the integral of $\rho$ over the region $x^2 + y^2 + z^2 \leq 4, \quad z \leq 1, \quad x \geq 0$. Draw a picture of the domain first!

   **Answer:** See 23 Oct solutions.

2. Find the volume of the [axisymmetric] region defined by rotating $z = 1/y$ about the $z$ axis and restricting $1 \leq z \leq a$. What happens as $a$ goes to infinity? [Hint: Rotating the curve $z = 1/y$ about the $z$ axis results in the surface $z = 1/r$.]

   **Answer:** See 23 Oct solutions.

3. Consider the conservative vector fields $F(x, y, z) = (x/r, y/r, z/r)$ and $G(x, y, z) = (x, y, z)$ and sketch representations of them in a plane. Find the potentials that induce these vector fields.

   **Answer:** In the text they spell out what the potentials for these vector fields are. To derive it ourselves, we suppose that there is a $V$ such that $\nabla V = F$, in which case $F_1 = \partial V/\partial x, F_2 = \partial V/\partial y$, and $F_3 = \partial V/\partial z$. Thus,

   $$V(x, y, z) = \int F_1 \, dx = \int (x^2 + y^2 + z^2)^{-1/2} \, x \, dx$$

   $$= \frac{1}{2} \left(\frac{(x^2 + y^2 + z^2)^{1/2}}{1/2}\right) + f(y, z)$$

   $$= r + f(y, z).$$

   Taking the partial of this with respect to $y$ we get $\partial V/\partial y = y/r + \partial f/\partial y$, and setting equal to $F_2$ we see $\partial f/\partial y = 0$. Similarly, we find $\partial f/\partial z = 0$. Therefore, $V(x, y, z) = r + \text{const.}$.

   In a similar fashion, $W(x, y, z) = \frac{1}{2}y^2 + \text{const.}$ can be shown to satisfy $\nabla W = G$.

4. Find the level surfaces (a.k.a. the equipotential surfaces) of the potentials found in the previous problem. Draw representations of them in a plane.

   **Answer:** $F(x, y, z)$ points away from the origin at all points $(x, y, z)$ and $\|F(x, y, z)\| = 1$ for all $(x, y, z)$. Since $F = \nabla V$, this means the gradient of $V$ points away from the origin at all points (which means the equipotential surfaces are spherical shells centered at the origin) and the gradient of $V$ has constant magnitude (meaning that the equipotential surfaces for $V = 1, 2, 3, ...$ are equally spaced). $G(x, y, z)$ also points away from the origin at all points $(x, y, z)$, but $\|G(x, y, z)\| = r$ at each $(x, y, z)$. The latter means that the magnitude of the gradient of $V$ is increasing as we get farther from the origin, i.e., the function $W$ is increasing faster as we get farther away from the origin. Hence, while the equipotential surfaces $W = 1, 2, 3, ...$ are also spherical shells just as they are for $V$, the equipotential surfaces for $W$ are getting closer and closer as we get farther from the origin. For example, the surfaces $W = 1$ and $W = 2$ are farther apart than the surfaces $W = 2$ and $W = 3$. 

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5. BONUS: What is the surface area of the region in problem 2 and what happens to the surface area as $a$ goes to infinity? This is very strange.

Answer: See 23 Oct solutions.