1. Give an example of a line with two parametrizations \( r_1(t) \) and \( r_2(t) \) for which there is no \( t \) such that \( r_1(t) = r_2(t) \).

**Answer:** Let \( r_1(t) = t \) and \( r_2(t) = t + 1 \). \( t = t + 1 \) has no solution. Keep in mind that if two parametrizations are given, their parameters are independent.

2. Find the intersection of the two planes:

   a) \( 6(x + 1/2) + 4(y + 1/2) + 8z = 0, \ 3x + 2y + 4z + 7 = 0 \).
   b) \( 6x + 4y + 8z + 5 = 0, \ 3x + y + 2z + 7 = 0 \).
   c) \( 6(x - 1) + 4(y - 3) + 8(z + 3) = 0, \ 9(x - 1) + 6(y + 1) + 12(z + 1) = 0 \).

   **Answer:** The general approach is taken and is as follows. Compute the cross product of the normal vectors. If the cross product is zero, then the planes are parallel, and one must determine if the two planes are the same or not. If the cross product is not zero, the cross product is a direction vector for a line of intersection, and a point on the line can be found by solving for a point on both planes. a) The planes never intersect. b) The cross product of the two vectors is \((0, 12, -6)\), and a point with \( z = 0 \) common to both planes is \((23/6, 9/2, 0)\). Therefore one parametrization for the line of intersection of the two planes is \((0, 12, -6)t + (-23/6, 9/2, 0)\). c) The two planes are the same.

3. Parametrize the curve \( \mathcal{C} \) obtained as the intersection of the surfaces \( x^2 - y^2 = z - 1 \) and \( x^2 + y^2 = 4 \).

   **Answer:** Example 3 in Section 14.1 in the text.

4. How would you parametrize a unit circle that is centered at \( P \) and lies in the plane orthogonal to \( N \)?

   **Answer:** What it first occurs to me to do is something out of the scope of this course, but is not so hard to follow. First of all, we can immediately say that the set of points \( R \) we are looking for must satisfy \((R - P) \cdot N = 0\) and \( \|R - P\| = 1 \). Now, take two orthogonal vectors \( u \) and \( v \) lying in the plane orthogonal to \( N \) (one way to find \( u \) and \( v \) is to take \( u = Q - P \), where \( Q \) is some other point in the plane, and then let \( v = u \times N \)). Next consider the curve \( c(t) = (\cos t)u/\|u\| + (\sin t)v/\|v\| + P \). I claim that this parametrization draws the desired figure. To see this, notice first that \((c(t) - P) \cdot N = 0\) because both \( u \) and \( v \) are orthogonal to \( N \):

\[
(c(t) - P) \cdot N = \left( \frac{\cos t}{\|u\|} u + \frac{\sin t}{\|v\|} v \right) \cdot N \\
= \frac{\cos t}{\|u\|} u \cdot N + \frac{\sin t}{\|v\|} v \cdot N \\
= 0.
\]
Next, notice that \( \|c(t) - P\| = 1 \) for all \( t \):

\[
\|c(t) - P\|^2 = (c(t) - P) \cdot (c(t) - P) \\
= (\cos tu / \|u\| + \sin tv / \|v\|) \cdot (\cos tu / \|u\| + \sin tv / \|v\|) \\
= \frac{\cos^2 t}{\|u\|^2} u \cdot u + \frac{\cos t \sin t}{\|u\| \|v\|} u \cdot v + \frac{\sin t \cos t}{\|v\| \|u\|} v \cdot u + \frac{\sin^2 t}{\|v\|^2} v \cdot v \\
= \cos^2 t + \sin^2 t \quad \text{(Using } u \cdot u = \|u\|^2 \text{ and } u \cdot v = 0) \\
= 1.
\]

This shows that \( c(t) \) is a parametrization of the unit circle centered at \( P \) and in the plane orthogonal to \( N \).

It is enlightening to think about the key points in the above proof and how it relates to the parametrizations of the circle in the \( xy \) plane. See me for further details if you wish.

5. Use considerations of the volume of the parallelepiped to show \( u \cdot (v \times w) = w \cdot (u \times v) = v \cdot (w \times u) \). (The handed-ness of the set \( u, v, w \) determines whether \( u \cdot (v \times w) \) is positive or negative, remember.)

Answer: This is discussed in the solution to problem 4 in the group work given 11 Sep.