Recall that Green’s theorem states: If $\mathcal{D}$ is a domain whose boundary $\partial \mathcal{D}$ is a simple closed curve, oriented counterclockwise, then

$$\oint_{\partial \mathcal{D}} F_1 \, dx + F_2 \, dy = \iint_{\mathcal{D}} \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \, dA.$$ 

1. Draw arrows indicating the boundary orientation for the region between two concentric circles.

   **Answer:** The inner boundary is oriented clockwise; if you walk clockwise along the inner boundary, the region is to your left, as desired. Similarly, the outer boundary is oriented counterclockwise.

2. Recall that $\int_{\mathcal{C}} \mathbf{F} \cdot \mathbf{ds}$ and $\int_{\mathcal{C}} F_1 \, dx + F_2 \, dy$ are two notations for the same thing. For which of the following vector fields do we have $\int_{\mathcal{C}} \mathbf{F} \cdot \mathbf{ds}$ equal to the area enclosed by $\mathcal{C}$ for every closed curve $\mathcal{C}$?

   a) $\mathbf{F} = \langle -y, 0 \rangle$
   
   b) $\mathbf{F} = \langle x, y \rangle$
   
   c) $\mathbf{F} = \langle \sin(x^2), x + e^{y^2} \rangle$

   **Answer:** We need $\partial F_2 / \partial x - \partial F_1 / \partial y = 1$ because in that case Green’s theorem says $\int_{\partial \mathcal{D}} \mathbf{F} \cdot \mathbf{ds} = \text{area of } \mathcal{D}$ for every region $\mathcal{D}$. This is clearly satisfied by a) and c) but not b).

3. Use Green’s theorem to find $\oint_{\mathcal{C}} x^2 \, dy$ where $\mathcal{C}$ is the unit circle centered at the origin.

   **Answer:** The vector field we’re concerned with is $\mathbf{F} = \langle x^2 y, 0 \rangle$. Then, $\partial F_2 / \partial x - \partial F_1 / \partial y = 0 - x^2$, and by Green’s theorem $\oint_{\mathcal{C}} x^2 \, dy = \iint_{\mathcal{D}} -x^2 \, dA$, where $\mathcal{D}$ is the unit disk centered at the origin. I found $\iint_{\mathcal{D}} -x^2 \, dA = -\pi/4$. 

1