MATH 1120 Calculus II - Fall 2018 22 Oct 2018

Problem Set 21

A local fishery is trying to improve its salmon hatcheries and have hired you to consult them for their second decade of operation. In the first decade, they built 4 identical salmon hatcheries seeded with a different amount of salmon juveniles. They found that if they filled a hatchery 25% by volume, by the end of the decade they would have a thriving salmon population that fills 40% of the hatchery. They also found that if they filled a hatchery to 90%, overcrowding would reduce the salmon population to 80% of the hatchery. In the third and fourth hatchery, they filled each respective pool halfway and three-quarters full. Both of these hatcheries yielded a salmon population that filled 70% of the hatchery pool.

a) Derive a differential equation that can be used to model this local fishery.

b) Draw a complete phase line diagram highlighting all equilibria, increasing and decreasing solutions, and solution concavity.

c) Plot all solutions that are possible for your model. Explain what each of these solutions mean for the local fishery?

A 1500 liter tank initially contains 600 liters of water with 5 kilograms of salt dissolved in it. Water enters the tank at a rate of 9 liters per minute and the water entering the tank has a salt concentration: $f(t) = \dfrac{1}{t^2 + 400t + 40001}$ kilograms per liter. Assume that a well-mixed solution leaves the tank at a rate of 6 liters per minute, how much salt is in the tank when it overflows?