

Problem Set 20

Let P be the percentage of a petri dish covered by bacterial colonies. Consider the following population model:

$$\frac{dP}{dt} = kP(2P - 1)(3P - 4)(P - 1), \quad P(0) = P_0$$

1) Set $k = 1$ and draw a phase line for this DE.

2) Identify equilibria and classify their stability.

3) Sketch solutions for this DE.

4) Consider $P_\infty = \lim_{t \rightarrow \infty} P(t)$ with initial value $P_0 = 1/2$. Determine P_∞ . What does this mean in the context of the problem?

5) What value(s) of k ensure that the bacterial thrives for any $P_0 \neq 0$?