

Problem Set 17

1) Solve the following differential equations:

(a) (7.2.14)

$$\sqrt{2xy} \frac{dy}{dx} = 1$$

(b) (7.2.16)

$$(\sec x) \frac{dy}{dx} = e^{y+\sin x}$$

(c) (7.2.18)

$$\frac{dy}{dx} = \frac{e^{2x-y}}{e^{x+y}}$$

(d) (7.2.20)

$$\frac{dy}{dx} = xy + 3x - 2y - 6$$

2) Consider the function: $y(x) = 100 - \frac{1}{5} \int_0^x (y(t) - 20) dt$.

(a) What initial value problem is this function the solution of?

(b) Express the solution $y(x)$ using explicit functions of x .

(c) If $y(x)$ describes the temperature in Celsius of a mug of coffee sitting on a countertop, how long would it take for the object to reach 50C?

(d) What temperature does the mug of coffee approach after a long period of time?

3) For each initial value problem, draw the associated slope field. Use Euler's method to calculate the first three approximations to the given initial value problem for the specified increment size. Calculate the exact solution and investigate the accuracy of your approximations. Round your results to four decimal places.

(a) $y' = 1 - \frac{y}{x}$, $y(2) = -1$, $dx = 0.5$.

(b) $y' = 2xy + 2y$, $y(0) = 3$, $dx = 0.2$.