

Prelim II Prep

- 1) Identify whether each statement is always true or sometimes false. Provide a counterexample if the statement is false or explain why the statement is true.

a) $\int_{-\infty}^{\infty} f(x) dx = \lim_{R \rightarrow \infty} \int_{-R}^R f(x) dx.$

- b) If a first-order differential equation has 3 equilibrium points, then at most two of these equilibria are stable.

2) Gabriel's Horn is a fairly quirky geometric object that is obtained by revolving the graph of $y = 1/x$ for $x > 0$ about the x -axis.

a) Determine the volume of this horn.

b) Determine the surface area of this horn.

3) (10.1.111) Newton's method is a way to numerically find the solutions to the equation $f(x) = 0$ and can be defined recursively:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Which of the following sequences converge? If so, to what value?

a)

$$x_0 = 1, \quad x_{n+1} = x_n - \frac{x_n^2 - 2}{2x_n}$$

b)

$$x_0 = 1, \quad x_{n+1} = x_n - \frac{\tan x_n - 1}{\sec^2 x_n}$$

c)

$$x_0 = 1, \quad x_{n+1} = x_n - 1$$

4) Determine if the following sequences converge. If so, determine its limit.

a)

$$a_n = \frac{2^n}{n!}$$

b)

$$b_n = (\ln n)^{1/n}$$