

## Prelim I Prep

1) Compute the area between the curve  $y = (x - 1)^2 - 1$  and the  $x$ -axis over the interval  $[0, 5]$ .

2) Let  $y = \lceil x \rceil$  denote the graph of the ceiling function, where  $y$  is the smallest integer larger than or equal to  $x$ . Compute the area of the region between the graph of this function and the  $x$ -axis over the interval  $[0, 5]$ .

3) Explain how one would go about computing a quantity like volume of a solid using definite integrals.

4) Let  $\mathcal{D}$  denote the equilateral triangle contained in the first quadrant with two vertices at the origin and at  $(2, 0)$ .

(a) Compute the area of the equilateral triangle using a definite integral.

(b) Let  $\mathcal{W}$  denote the solid of revolution obtained by revolving  $\mathcal{D}$  about the  $y$ -axis. Represent the total surface area of  $\mathcal{W}$  using definite integrals.

(c) Compute the volume of  $\mathcal{W}$  using cross-sections perpendicular to the axis of revolution.

(d) Compute the volume of  $\mathcal{W}$  using cross-sections parallel to the axis of revolution.

- (e) A tank in the shape of the solid  $\mathcal{W}$  is filled completely with water. How much work is done to pump this water from ground level ( $y = 0$ ) into the tank? Let  $w$  be the density of water and express your answer in terms of  $w$ .

- (f) Compute the volume of the solid whose cross-sections perpendicular to the plane are equilateral triangles with bases that run parallel to the  $x$ -axis.