ESMI Applied Math - Summer 2019 27 Jun 2019

The NYC Taxi Model

Expected Skills.
Students can...

- *encode Markov models in MATLAB.*
- *identify and analyze Markov models for steady state solution.*
- *contextualize mathematical statements to conclusions about the model.*

Guiding Questions

(a) What are the identifying characteristics of a Markov model?

(b) What are some useful built-in functions of MATLAB to generate time simulations?

Encoding the NYC Taxi Model

The movement of NYC taxi cabs can be modeled via a Markov chain model, where the distribution of taxi cabs is tracked as it evolves due to some transition mechanism. We can reduce each borough of NYC as a bin with lanes that allow for cabs to travel between each borough. Let u be a vector with 5 coordinates that represent the fraction of the whole NYC taxi fleet that lie within a specific borough of NYC. Specifically:

- `u(1)` refers to taxis in Manhattan,
- `u(2)` refers to taxis in Brooklyn,
- `u(3)` refers to taxis in Queens,
- `u(4)` refers to taxis in the Bronx, and
- `u(5)` refers to taxis in Staten Island.

How this distribution of taxis changes hourly can be represented by the transition matrix A:

```
A = [
    0.6500    0.2500    0.2500    0.4000    0.4700
    0.1000    0.5000    0.2000    0.0500    0.0100
    0.1000    0.2000    0.5000    0.0500    0.0100
    0.1000    0.0250    0.0500    0.5000    0.0100
    0.0500    0.0250    0.0000    0.0000    0.5000
    ];
```

Also known as a Markov or stochastic matrix, this matrix has the special property that its columns sum to 1 and facilitates the transition to the next state by left multiplication of the current state. Repeated multiplication allows us to generate sequence of states that track the distribution of taxi cabs over the course of several hours. Hence we have our NYC Taxi Model:

$$u_{n+1} = Au_n.$$

Analyzing the Taxi Cab Model

(a) How should you interpret the state $u = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$?

(b) How should you interpret the entries of the matrix A? It may be helpful to consider the relationship between u_{n+1} and u_n.

(c) Compute the sequence of states from a random initial distribution of taxis u_0 to u_{20} and plot this sequence against time. Explain the resulting figure you generated. Repeat these steps for other initial distributions of taxis.

(d) Using the built-in function `eig` determine the eigenvalues and eigenvectors of the matrix A. How do the eigenvalues and eigenvectors correlate to your long-term average solution?

(e) Does this model seem realistic? What changes to your model would you make to better the realism?

Project Extensions

There are various ways to extend and pivot from Markov chain models:

- Consider time-varying transition probabilities in the Markov matrix A,
- Hidden Markov Models, Hidden Variables, and Mixture Models where hidden sub-populations impact observable population states,
- Repeated Games such as the repeated prisoner's dilemma or rock-paper-scissors,
- Queueing Theory, and
- Monte Carlo Method and its application to Numerical Integration.