ESMI Applied Math - Summer 2019 | 21 Jun 2019

The Logistic Map

Expected Skills.
Students can...

- *encode iterative maps in MATLAB.*
- *generate orbits of maps in MATLAB.*
- *identify and classify equilibria of maps.*
- *perform parameter studies of ODE models in MATLAB.*
- *contextualize mathematical statements to conclusions about the model.*

Guiding Questions

(a) What is the general workflow of implementing a discrete-change system in MATLAB?

(b) What are some useful built-in functions of MATLAB to generate time simulations?

(c) What are equilibria and how do you go about classifying them for discrete models?

Encoding the Logistic Model

The Logistic Model is an iterative map that has been used to model population growth for a species subject to a carrying capacity. The model is a series of discrete jumps and is given by the following map:

$$x_{n+1} = F(x_n, a) = rx_n(1 - x_n).$$

- Within the context of population modeling, what does the variable represent? What values can the the variable meaningfully take on? What does the parameter represent?
- Encode the map: `xnew = logistic(xold,r)`. What does this function do?

Analyzing the Logistic Model

The jumps that the population take as we iterate the logistic map form an *orbit* that can be seen as the population's rise and decline across years.

(a) Set $r = 2.5$. Plot the orbit when the population starts at $x_0 = 0.5$. Compare this orbit with other orbits generated by other initial populations.

(b) Are there any equilibria of this logistic map? Classify any that you find.

(c) Set $r = 3.2$ and repeat (a) and (b). Characterize any noteworthy orbits that you find.

(d) To investigate the impact of the parameter a, we plot large iterates of the interval against the parameter. Explain in your own words the following code snippet:

```
plot_min = 0;
plot_max = 1;
plot_points = 100;
num_iterations = 1e5;
end_sample = 10;

figure(1)
clf
set(gcf, 'position', [50 50 900 950])
set(gcf, 'PaperPositionMode','auto')
axis square
grid on
box on
set(gca, 'FontSize', 14)
hold on

r_vec = linspace(2.5, 4, 300);
for r = r_vec
    x = linspace(plot_min, plot_max, plot_points);
    for i = 1:num_iterations
        x = r .* (1-x) .* x;
        if ((num_iterations - i) < end_sample)
            plot( r*ones(1,plot_points), x, '.k', 'MarkerSize', 4)
        end
    end
end
hold off
xlim([2.5, 4])
ylim([0, 1])
title('Route to Chaos: $x_{n+1} = rx_n(1-x_n)$', ...
    'FontSize', 22, ...
    'Interpreter', 'latex')
xlabel('$r$',  'FontSize', 22, 'Interpreter', 'latex')
ylabel('$x$',  'FontSize', 22, 'Interpreter', 'latex')

saveas(gcf,'logistic_bifurcation.pdf')
```

(e) Run the code snippet and explain in your own words the figure that is generated. What do you notice as r increases?