Initial Work on Formalizing the Multiple-Time Scale Structure of Power System Models

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Introduction

Driven by the need for more accurate representations of modern electric grids, power system models are incorporating more quantities that evolve on vastly different time-scales. One manifestation of this arises in power systems interfacing with high voltage direct current systems. The common approach is to perform a hybrid or tandem simulation of a transient stability model and an electromagnetic model. The work here proposes a multiple time scale formalism for interfacing these two models and analyzes the framework numerically using two power systems: a 3-bus test grid and the 2224-bus Great Britain transmission network.

Formal Model

Full Model

- Three variables on separate explicit time scales:
    - x: slow, mechanical (swing angle, frequency)
    - y: fast, electromechanical (q-axis and d-axis voltages, voltage exciters)
    - z: hyperfast, electromagnetic (network voltages and currents)
- Two explicit time scale parameters:
    - ε: separation (ratio) of time scales for slow and fast states
    - η: separation (ratio) of time scales for fast and hyperfast states
- Assumption: hyperfast states reduce any network equations to pure differential equations (DEs).

$$\begin{aligned} \dot{x} &= \varepsilon f(x,y,z) \\ \dot{y} &= g(x,y,z) \\ \eta\dot{z} &= h(x,y,z) \end{aligned}$$

Subsystems

- Slow subsystem:
    - Rescale time: $t \to t/\varepsilon$, then consider the limit: $\varepsilon \to 0$
    - Fast and hyperfast vector fields can be simplified to algebraic constraints.
    - Constrained DE: dynamic x, algebraic (y,z)

$$\begin{aligned} \dot{x} &= f(x,y,z) \\ 0 &= \hat{g}(x,y,z) \\ 0 &= \hat{h}(x,y,z) \end{aligned}$$

- Fast subsystem:
    - Consider the limit: $\varepsilon \to 0$
    - Slow variables become constant and hyperfast vector fields can be simplified to algebraic constraints.
    - Parameterized, constrained DE: dynamic y, algebraic z, constant x

$$\begin{aligned} \dot{x} &= 0 \\ \dot{y} &= g(x,y,z) \\ 0 &= \hat{h}(x,y,z) \end{aligned}$$

- Hyperfast subsystem:
    - Rescale time: $t \to \eta t$, then consider the limit: $\eta \to 0$
    - Slow and fast variables become constant.
    - Parameterized DE: dynamic y, algebraic z, constant x

$$\begin{aligned} \dot{x} &= 0 \\ \dot{y} &= 0 \\ \dot{z} &= \hat{h}(x,y,z) \end{aligned}$$

Time Simulations

Integrator

- Trajectories of the full model can be approximated by **Candidate Trajectories**
- Candidate trajectories are concatenations of subsystem trajectories.
- Subsystem trajectories can be computed
    - using efficient numerical methods suited to the structure of the subsystem
    - quickly due to high dimensionality reduction
- Subsystem switching depends upon regularity of constraining algebraic equations
    - Singular equations → switch to faster subsystem
    - Equilibria in subsystem → switch to slower subsystem

Implementation

- Add-on package to Federico Milano's Power System Analysis Toolbox (PSAT)
- Parallelizable using MatLab's Parallel Computing Toolbox
- Constrained DEs solved using an Implicit Trapezoidal Rule
- Pure DEs solved using an 8th order Dormand-Prince method.
- Results obtained using a computer operating with an Intel(R) Core(TM) i7-6700HQ processor and GeForce GTX 960M GPU

Trajectories

- Performance gains are apparent with large power systems
- Candidate trajectories have more accurate state projections of faster time-scales

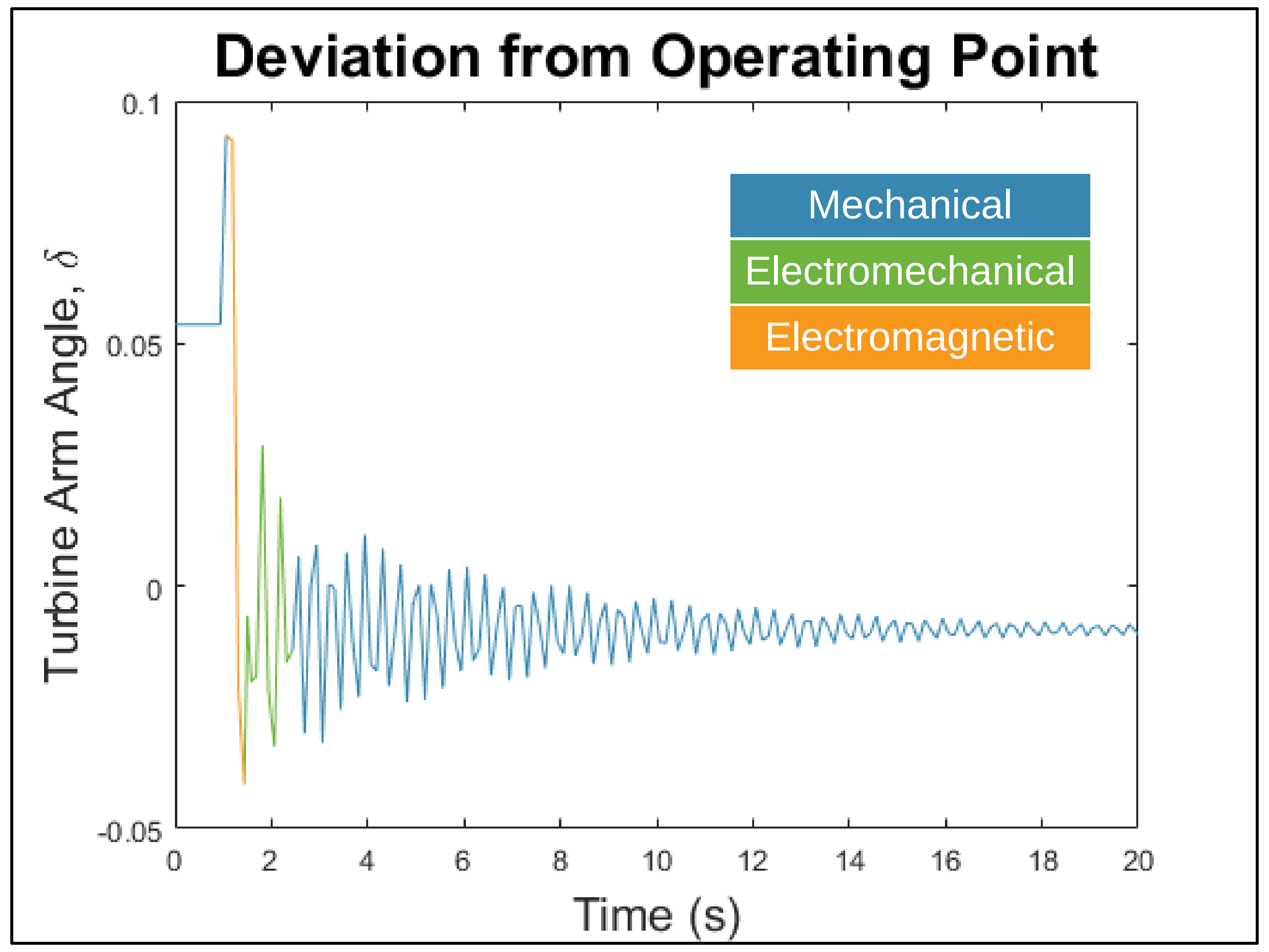

Fig. 1: Generator response to perturbation of 3-bus microgrid.

Test Cases

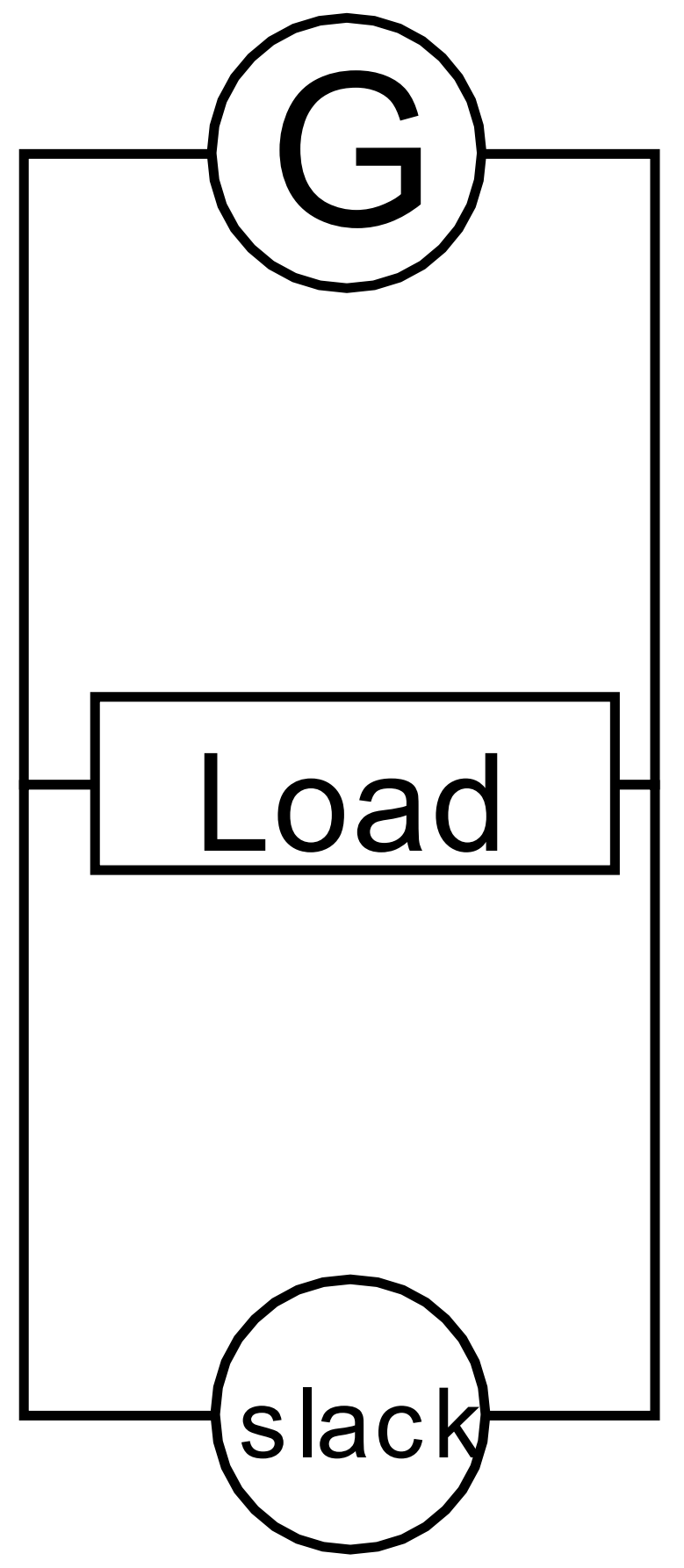

Fig. 2: The 3-bus microgrid.

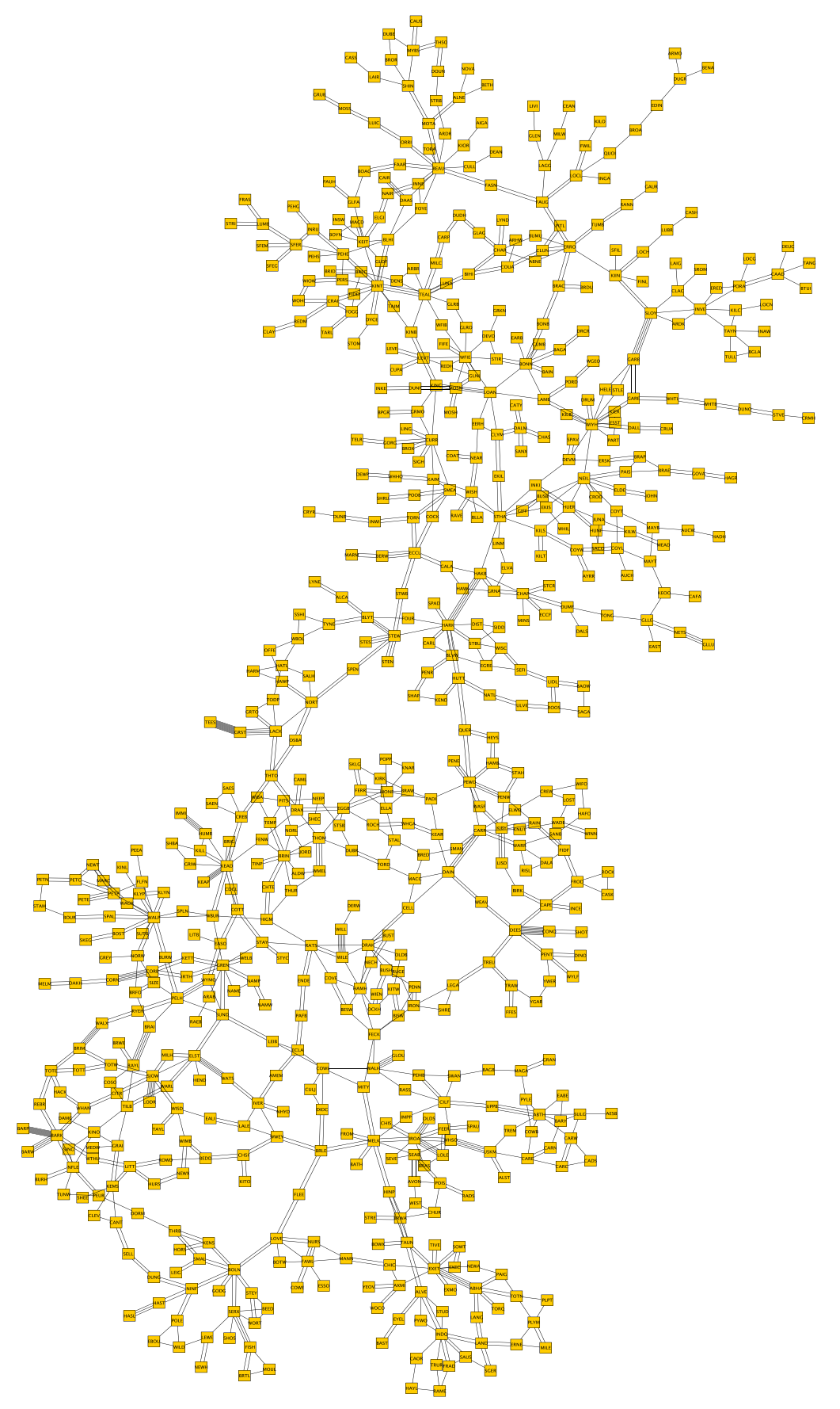

Fig. 3: The 2224-bus Great Britain transmission network.

- Third-order synchronous machines
- Generators regulated by nonlinear voltage exciter
- Constant power demands at loads
- First-order induction motors at loads
- Low-pass time-varying phasor representation of electromagnetic states

Future Work

- Phenomenological study of power systems through formal model
- Increase computational performance (local Y-bus updates, improve matrix construction, etc.)
- Extension to long-term stability models (inclusion of slow discrete subsystem)

References

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