MA 513
Test 2 Study Guide

Spring 2010

Geometric series - We say

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1-z} \quad \text{whenever} \quad |z| < 1$$

Taylor series - Let f be analytic throughout $|z - z_0| < R_0$, centered at z_0 with radius R_0. Then $f(z)$ has the power series representation

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n \quad (|z - z_0| < R_0)$$

where

$$a_n = \frac{f^{(n)}(z_0)}{n!} \quad (n = 0, 1, 2, \ldots)$$

Common Formulas

$$\begin{aligned} e^z &= \sum_{n=0}^{\infty} \frac{z^n}{n!} \\ \sin z &= \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!} \\ \cos z &= \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!} \end{aligned}$$

Laurent Series - Suppose that a function f is analytic throughout an annular domain $R_1 < |z - z_0| < R_2$, centered at z_0, and let C denote any positively oriented simple closed contour around z_0 and lying in that domain. Then, at each point in the domain, $f(z)$ has the series representation

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z - z_0)^n} \qquad (R_1 < |z - z_0| < R_2)$$

where

$$\begin{aligned} a_n &= \frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^{n+1}} dz \qquad (n = 0, 1, 2, \ldots) \\ b_n &= \frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^{-n+1}} dz \qquad (n = 1, 2, \ldots) \end{aligned}$$